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DERIVING FUNCTION-FAILURE SIMILARITY INFORMATION FOR FAILURE-FREE ROTORCRAFT COMPONENT DESIGN

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ABSTRACT

Performance and safety are the top concerns of high-risk aerospace applications at NASA. Eliminating or reducing performance and safety problems can be achieved with a thorough understanding of potential failure modes in the designs that lead to these problems. The majority of techniques use prior knowledge and experience as well as Failure Modes and Effects as methods to determine potential failure modes of aircraft. During the design of aircraft, a general technique is needed to ensure that every potential failure mode is considered, while avoiding spending time on improbable failure modes. In this work, this is accomplished by mapping failure modes to specific components, which are described by their functionality. The failure modes are then linked to the basic functions that are carried within the components of the aircraft. Using this technique, designers can examine the basic functions, and select appropriate analyses to eliminate or design out the potential failure modes. This method was previously applied to a simple rotating machine test rig with basic functions that are common to a rotorcraft. In this paper, this technique is applied to the engine and power train of a rotorcraft, using failures and functions obtained from accident reports and engineering drawings.

KEYWORDS

Failure analysis; Functional modeling; Function-failure commonality; Functional decomposition for product design; Failure-free component design.

INTRODUCTION

Failures in aircraft components in high-risk applications are unacceptable in terms of safety and performance. In this work, methods of recording, understanding, and predicting failure modes are regarded to be essential to advance the field of fault monitoring and failure prevention [1-4]. In designing a new product or redesigning an existing product, designers often draw similarities between the new product and other related products [5]. This provides the designer with possible failure modes that may occur in the parts of the new design through experience with the similar designs. Unfortunately this does not supply all possible failure modes. It is generally not possible to analyze all possible failure modes that could occur in the new design only through comparisons with similar products. Designers need a fundamental way to capture and interpret past failures and utilize that information in the new design.

To help with this goal, the fundamentals of a design-aid tool were presented by Tumer and Stone in [6] to explore the connection between failure modes and the functionality of components and form a tool that designers may use to understand and prevent failures during conceptual and embodiment design. Once this correlation between failure modes and functionality of the components is established, then component solutions for each function can synthesized and designed to eliminate or significantly reduce a known failure modes [6].

The focus of this paper is to decompose realistic products, in this case a rotorcraft, into their basic components and then decompose the components into their functionality. We hold that components have a "commonality" at some basic level in terms of their functionality and failure modes. The common modes of failure can be determined once the functionality of the component or product is established. Once these failure modes are paired to these basic functions, then a larger family of components and systems can be considered. Using this generalization, this work formalizes the process of feeding back

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failure mode and reliability information into design and manufacturing phases by transforming the information into a form that can be used effectively by engineers [6-8].

APPLICATION: ROTORCRAFT COMPONENT FAILURES

Helicopters have been a major safety concern to all types of agencies that use them for everyday operations. Despite the maturity of helicopter technology, the probability of fatal accidents in rotorcraft is higher than in other aircraft [9]. The preservation of human life is NASA's number one concern. To address this concern, it is necessary to expose potential failures modes that could occur during operation early in the design stages in order to reduce the chances of failure. In this paper, the engine and power train of a Bell 206 helicopter were studied for this purpose. Diagrams of the compressor, gas producer, and power turbine assemblies of the Allison 250 engine are presented for reference in Figure 1a, 1b, and 1c, respectively [10].

To examine and gather data from the research helicopters used in the rotorcraft division, NASA Ames Research Center was visited in July of 2001. The Bell OH-58A was one the helicopters that was examined, which is the sister military model of the Bell 206 civilian model. The OH-58A at NASA Ames was a research helicopter partially used for failure analysis through monitoring vibration and noise signal behaviors [11]. Communication with Major David R. Arterburn provided information on the systems and maintenance of systems within the OH-58A helicopter [12]. Finally, accident reports published by the National Transportation Safety Board (NTSB) were studied thoroughly to extract common failure modes [9, 13].

There were 29 components and subsystems that were identified in the Bell 206 turbine engine and power train. In the NTSB reports, there were ten different types of failure modes recorded for these 29 components by the NTSB since 1983 [13]. The failure mode data gathered from the NTSB reports with respect to the components was formed into a matrix that is used in matrix manipulations to create design tools as described later in this paper [12]. In particular, there were 1000 reports that involved the Bell 206. The reports were reviewed and all reports with component failures for the engine and power train were noted. There were 69 cases of component failures for the engine and power train recorded. The remaining reports mostly consisted of error in pilot judgment. Some examples are misjudgment in fuel reserves, forgetting to detach all tie downs, collisions into power lines, and fuel contamination. Most of the pilot error reports could be traced to carelessness, which could be addressed by better training and procedures.

FUNCTION-FAILURE METHOD: A DESIGN TOOL

The function-failure method is based on previous work by Stone et al., Little et al. and McAdams et al. to derive the similarity between different designs based on functionality and to provide a repository of product design knowledge for
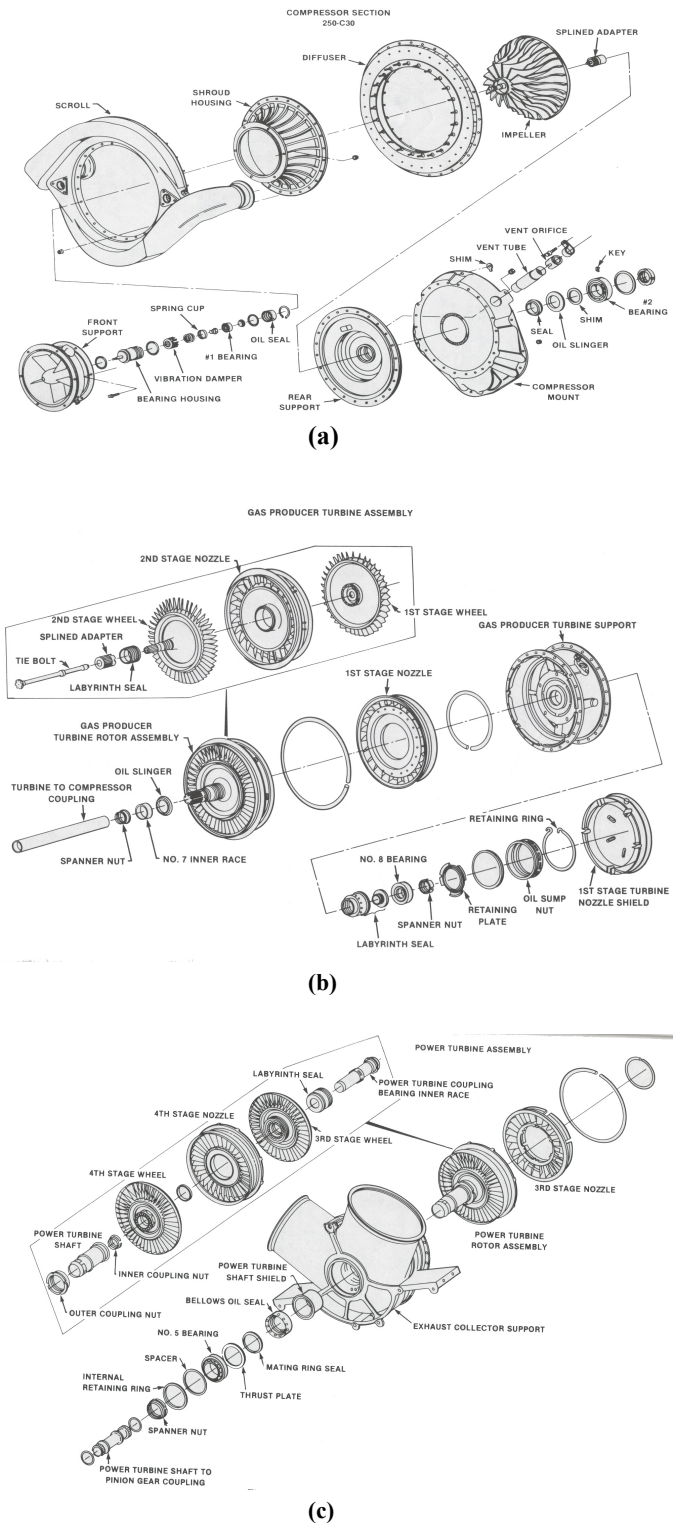
(a)

(b)

(c)

Figure 1. (a) compressor assembly, (b) gas producer assembly, and (c) power turbine assembly of the Allison 250 engine

Table 1. Failure vector

Failure	
F1 :	*bond failure*
F2 :	*corrosion*
F3 :	*fatigue*
F4 :	*fracture*
F5 :	*fretting*
F6 :	*galling and siezure*
F7 :	*human*
F8 :	*stress rupture*
F9 :	*thermal shock*
F10 :	*wear*

Table 2. Component vector

Component	
C1 :	*air discharge tubes*
C2 :	*bearing*
C3 :	*bleed valve*
C4 :	*bolt*
C5 :	*compressor case*
C6 :	*compressor mount*
C7 :	*compressor wheel*
C8 :	*coupling*
C9 :	*diffuser scroll*
C10 :	*exhaust collector*
C11 :	*fire wall*
C12 :	*front diffuser*
C13 :	*front support*
C14 :	*governor*
C15 :	*housing*
C16 :	*impeller*
C17 :	*mount*
C18 :	*nozzle*
C19 :	*nozzle sheild*
C20 :	*'O' ring*
C21 :	*P3 line*
C22 :	*plastic lining*
C23 :	*pressure control line*
C24 :	*pylon isolater mount*
C25 :	*rear diffuser*
C26 :	*rotor*
C27 :	*shaft*
C28 :	*spur adapter gearshaft*
C29 :	*turbine wheel*

designers [8, 14, 17, 18]. In this work, the function-failure method is applied as a design-aid tool that extends the idea of similarities to failure detection for rotorcraft components [6, 11]. Once failure modes in high-risk aerospace applications have been linked to the functionality of components, the designer can draw conclusions on how to design or redesign the components. Early in the design stage the components can be altered to be less susceptible to the failure mode. If possible, the component can be replaced by another component that performs the similar functions, but is not affected by the failure mode at hand.

Preliminary Matrix Computations

The engine and power train of the helicopter were broken down into components and subsystems. Let **C** be a 29 x 1 vector of the sub-systems or components of the engine and power train. Let **F** be a 10 x 1 vector of the failures modes that were found in NTSB accident reports involving the Bell 206 helicopter that have occurred since 1983. Vectors **F** and **C** are found in Tables 1 and 2 respectively.

The failure information is represented by weaving the individual vectors (containing information on failure modes, functionality and components) into matrices of information useful for computation. The failure modes are recorded with respect to components and subsystems. In the component-failure matrix **CF**, the rows represent the components and columns represent the failure modes. The matrix **CF** is found in Table 3 in binary form. A "1" is given if the failure mode occurred for the component and a "0" otherwise. Figure 2 provides a more visual representation of the component-failure mode data. The matrix from Table 3 was used to construct the chart.

Next, the functional model for the components of the engine and the power train are derived. An example of a function chain for the compressor wheel is shown in Figure 3. The compressor wheel performs three functions: change gas, convert mechanical energy to pneumatic energy, and guide gas. A more complete description of functional modeling is presented in Stone et al [7, 8] and Hirtz et al [16]. Let **E** be a 25×1 vector containing the elemental functions and their flows describing the components of the engine and power train.

Vector **E** is found in Table 4. A matrix was constructed by weaving vector **E** with **C**. The functions are represented in the rows and the components are represented in the columns. The function-component matrix (**EC**) is shown in Table 5. For the rows of the matrix, the energy flows of the functions are mechanical energy = me, thermal energy = th, pneumatic energy = pn. The elements in the matrix provide information for what function each component performs. The matrix is in binary form. A "1" is given if the component performs the function and a "0" otherwise. The **EC** is similar to the product-function matrix **Φ** found in previous work [8], except that **EC** gives information about the functionality of the components rather than the entire product. Once the component-failure and the function-component matrix are constructed, the function-failure matrix, **EF,** can be computed as:

$$\mathbf{EF} = \mathbf{EC} \times \mathbf{CF} \tag{1}$$

Table 3. Component-failure mode matrix CF

	bond failure (F1)	corrosion (F2)	fatigue (F3)	fracture (F4)	fretting (F5)	galling and siezure (F6)	human (F7)	stress rupture (F8)	thermal shock (F9)	wear (F10)
C1 : air discharge tubes	0	0	0	0	0	0	0	0	0	0
C2 : bearing	0	0	1	0	0	1	0	0	1	1
C3 : bleed valve	0	0	0	0	0	0	0	0	0	0
C4 : bolt	0	0	1	0	1	0	0	0	0	1
C5 : compressor case	0	1	0	0	0	0	0	0	0	0
C6 : compressor mount	0	0	1	0	0	0	0	0	0	0
C7 : compressor wheel	0	0	1	0	0	0	0	1	1	0
C8 : coupling	0	0	1	0	0	0	0	0	0	1
C9 : diffuser scroll	0	0	0	0	0	0	0	0	0	0
C10 : exhaust collector	0	0	0	0	0	0	0	0	0	0
C11 : fire wall	0	0	0	0	0	0	0	0	0	0
C12 : front diffuser	0	0	0	0	0	0	0	0	0	0
C13 : front support	0	0	0	0	0	0	0	0	0	0
C14 : governor	0	0	1	1	0	0	0	0	0	1
C15 : housing	0	0	0	0	0	0	0	0	1	1
C16 : impeller	0	0	0	0	0	0	0	0	0	0
C17 : mount	0	0	1	0	0	0	0	0	0	0
C18 : nozzle	0	0	0	0	0	0	0	0	0	0
C19 : nozzle sheild	0	0	0	0	0	0	0	0	0	0
C20 : 'O' ring	0	0	0	0	0	0	1	0	0	1
C21 : P3 line	0	0	0	0	0	0	0	0	0	1
C22 : plastic lining	0	0	0	0	0	0	0	0	0	0
C23 : pressure control line	0	0	1	0	1	0	0	0	0	0
C24 : pylon isolater mount	0	0	0	0	0	0	0	0	0	1
C25 : rear diffuser	0	0	0	0	0	0	0	0	0	0
C26 : rotor	0	1	0	0	0	0	0	0	0	0
C27 : shaft	1	1	0	0	0	0	0	0	1	0
C28 : spur adapter gearshaft	0	0	1	0	0	0	0	0	0	0
C29 : turbine wheel	0	0	1	0	0	0	0	1	1	0

Table 4. Functionality vector E

E1	:	*change gas*
E2	:	*change th*
E3	:	*convert me to pn*
E4	:	*convert th to pn*
E5	:	*couple me*
E6	:	*couple solid*
E7	:	*distribute gas*
E8	:	*export gas*
E9	:	*guide gas*
E10	:	*guide solid*
E11	:	*import gas*
E12	:	*regulate gas*
E13	:	*regulate liquid*
E14	:	*regulate me*
E15	:	*secure solid*
E16	:	*stop liquid*
E17	:	*stop me*
E18	:	*stop mixture*
E19	:	*stop solid*
E20	:	*stop th*
E21	:	*store gas*
E22	:	*store solid*
E23	:	*transfer gas*
E24	:	*transfer me*
E25	:	*transfer pn*

The function-failure matrix is shown in Table 6. Matlab was used to perform the computations to find the function-failure matrix. The elements in **EF** relate the failure modes to the elemental functions. Each element ef_{ij} indicates how many components solving the function presented by the ith row experience the failure mode represented in the jth column.

When designing a new product, or in this case a new design for an engine or power train of a rotorcraft, the designer constructs a function-component matrix for the design **EC**. This essentially is a morphological representation of the component solutions to each function. The function-failure matrix **EF** storing the data gathered from previous designs is cross-multiplied by the transpose of the function-component matrix **EC** of the new product to obtain a component-failure matrix for the new product, defined as:

$$\mathbf{CF} = \mathbf{EC}^{\mathrm{T}} \times \mathbf{EF} \qquad (2)$$

This gives **CF**, the component-failure matrix, which provides the possible failures that a component may experience during operation. This allows the designer to select and perform the appropriate analyses for the failure modes or change out components to eliminate or reduce the failure modes early in the design stages before the components are given final form.

A more visual representation of **EF** can be seen in Figure 4. The chart gives a faster method of identifying the function to failure mode relationship. Note that the function 'secure solid' accounts for the most failures occurring in components.

Capturing Similarity for Design and Redesign

Other matrix manipulations of the data may be done to obtain additional design information. These manipulations result in similarity matrices, which provide designers with a tool to account for and design against potential failure modes. There are several different types of similarity matrices. The needs of the designer will determine which way is most useful. Each of the similarity matrices may be derived from the preceding component-function and component-failure matrices.
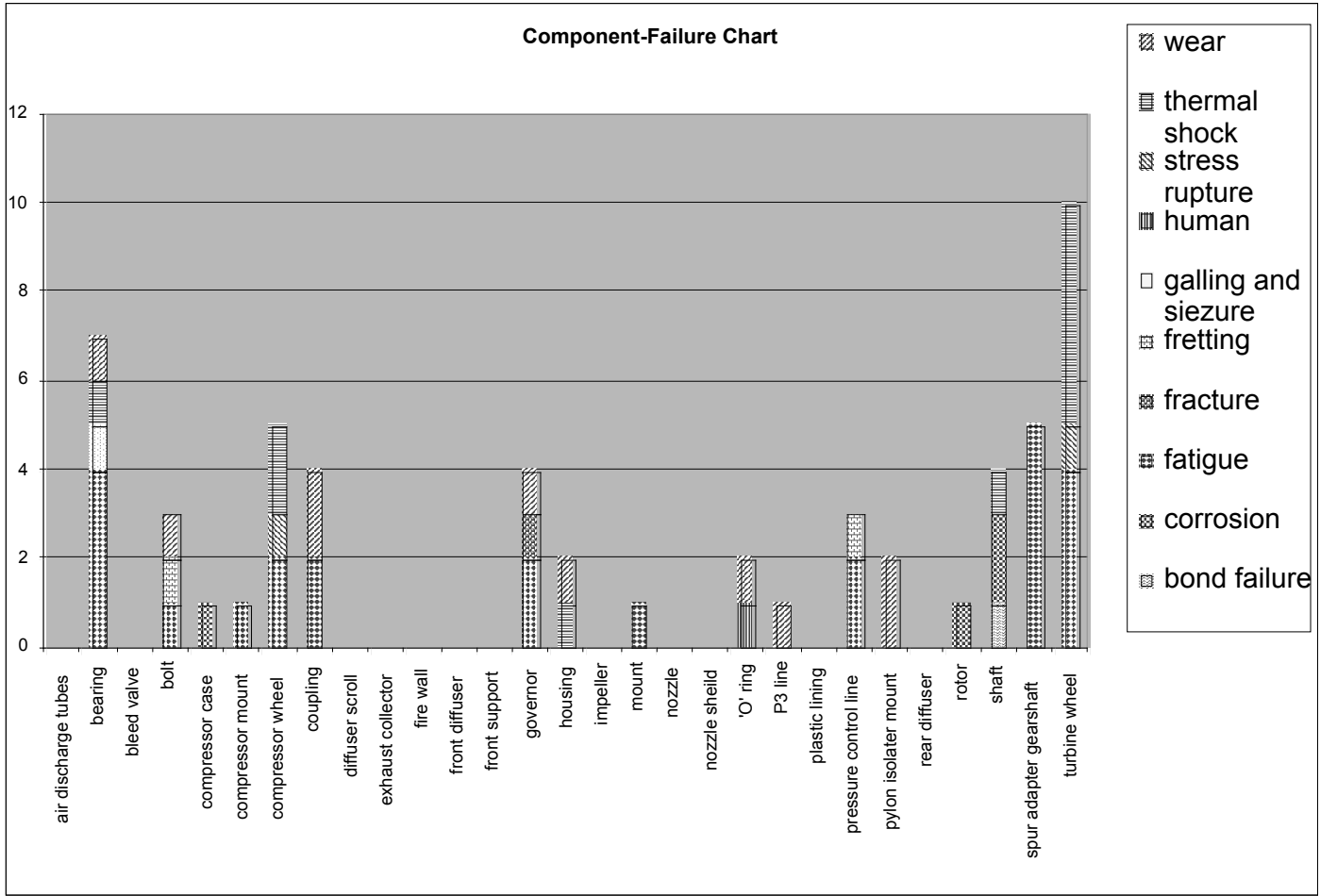
Figure 2. Bar chart of the Component-Failure matrix

First, consider the component-function similarity matrix $\hat{\Lambda}_{CE}$. Here we transpose the component-function matrix and post-multiply it by itself. This gives an m x m ($m = 29$) symmetric matrix. Mathematically, the component-function similarity matrix is defined as,

$$\hat{\Lambda}_{CE} = \overline{\mathbf{EC}}^{T} \times \overline{\mathbf{EC}} \qquad (3)$$

where $\overline{\mathbf{EC}}$ is the normalized matrix of the component-function matrix $\mathbf{EC}$, with the columns normalized to unity. The component-function matrix is normalized for convenience. Normalizing $\mathbf{EC}$ allows the similarity matrix to contain values between 0.0 and 1.0. Each of the elements $\hat{\lambda}_{CEij}$ represents the similarity between the components i and j based on the

elementary functions. The diagonal ($i = j$) is all ones because the component is completely similar with itself. Similarly, if the value is '1.0' elsewhere, then the two components are completely similar to each other, and if the value is '0.0', then the two components have no similarity (they do not share common elemental functions).

Next, the component-failure similarity matrix $\hat{\Lambda}_{CF}$ is computed from the component-failure matrix with its rows normalized to unity, $\overline{\mathbf{CF}}_C$, where the subscript C is added to emphasize that the rows or components are normalized. The component-failure similarity matrix is defined as:

$$\hat{\Lambda}_{CF} = \overline{\mathbf{CF}_{\mathbf{C}}} \times \overline{\mathbf{CF}_{\mathbf{C}}}^{\mathbf{T}} \qquad (4)$$

5

Table 5. The function-component matrix, EC

		C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18	C19	C20	C21	C22	C23	C24	C25	C26	C27	C28	C29
		air discharge tubes	bearing	bleed valve	bolt	compressor case	compressor mount	compressor wheel	coupling	diffuser scroll	exhaust collector	fire wall	front diffuser	front support	governor	housing	impeller	mount	nozzle	nozzle sheild	'O' ring	P3 line	plastic lining	pressure control line	pylon isolater mount	rear diffuser	rotor	shaft	spur adapter gearshaft	turbine wheel
E1 *change gas*	:	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	1	0	0	1
E2 *change th*	:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E3 *convert me to pn*	:	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
E4 *convert th to pn*	:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
E5 *couple me*	:	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0
E6 *couple solid*	:	0	0	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E7 *distribute gas*	:	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E8 *export gas*	:	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E9 *guide gas*	:	0	0	0	0	0	0	1	0	0	0	0	1	1	0	0	1	0	1	0	0	0	0	0	0	1	0	0	0	1
E10 *guide solid*	:	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E11 *import gas*	:	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E12 *regulate gas*	:	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
E13 *regulate liquid*	:	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E14 *regulate me*	:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
E15 *secure solid*	:	0	1	0	1	1	1	0	0	0	0	1	1	0	1	0	1	0	0	0	0	0	0	1	1	0	0	0	0	0
E16 *stop liquid*	:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
E17 *stop me*	:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E18 *stop mixture*	:	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E19 *stop solid*	:	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E20 *stop th*	:	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
E21 *store gas*	:	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E22 *store solid*	:	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E23 *transfer gas*	:	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
E24 *transfer me*	:	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0
E25 *transfer pn*	:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
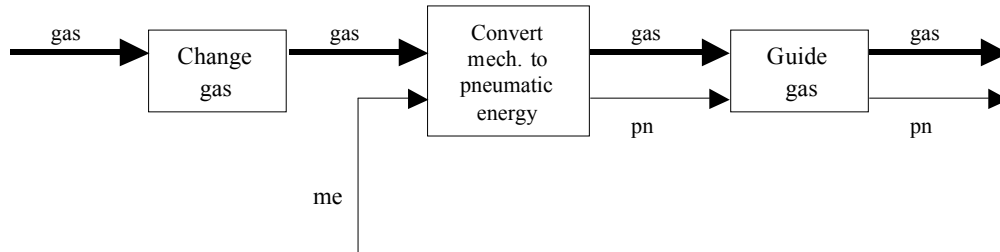
Figure 3. Function chain for a compressor wheel

The elements of $\hat{\Lambda}_{CF}$ indicate similarity between the components with respect to the failure modes they experience. The diagonal simply returns '1.0' since a component has the same potential failure modes as itself.

Finally, the similarity matrix for the failure-component matrix is also computed from the normalized component-failure mode matrix, $\overline{\mathbf{CF}}_F$, but the columns are normalized to unity. Again, the subscript F is added to denote that the columns or failure modes are normalized. Failure mode-component similarity is calculated as:

$$\hat{\Lambda}_{FC} = \overline{\mathbf{CF}}_F{}^T \times \overline{\mathbf{CF}}_F \tag{5}$$

Table 6. Function-failure matrix (EF).

			F1 bond failure	F2 corrosion	F3 fatigue	F4 fracture	F5 fretting	F6 galling and siezure	F7 human	F8 stress rupture	F9 thermal shock	F10 wear
E1	:	change gas	0	1	2	0	0	0	0	2	2	0
E2	:	change th	0	0	0	0	0	0	0	0	0	0
E3	:	convert me to pn	0	0	1	0	0	0	0	1	1	0
E4	:	convert th to pn	0	0	1	0	0	0	0	1	1	0
E5	:	couple me	1	1	2	0	0	0	0	0	1	1
E6	:	couple solid	0	0	3	0	1	0	0	0	0	2
E7	:	distribute gas	0	0	0	0	0	0	0	0	0	0
E8	:	export gas	0	0	0	0	0	0	0	0	0	0
E9	:	guide gas	0	0	2	0	0	0	0	2	2	0
E10	:	guide solid	0	0	1	0	0	1	0	0	1	1
E11	:	import gas	0	0	0	0	0	0	0	0	0	0
E12	:	regulate gas	0	0	0	0	0	0	0	0	0	0
E13	:	regulate liquid	0	0	1	1	0	0	0	0	0	1
E14	:	regulate me	0	0	0	0	0	0	0	0	0	1
E15	:	secure solid	0	1	4	0	1	1	0	0	2	4
E16	:	stop liquid	0	0	0	0	0	0	1	0	0	1
E17	:	stop me	0	0	0	0	0	0	0	0	0	0
E18	:	stop mixture	0	0	0	0	0	0	0	0	1	1
E19	:	stop solid	0	0	0	0	0	0	0	0	0	0
E20	:	stop th	0	0	1	0	0	1	0	0	1	1
E21	:	store gas	0	0	0	0	0	0	0	0	0	0
E22	:	store solid	0	0	0	0	0	0	0	0	1	1
E23	:	transfer gas	0	0	1	0	1	0	0	0	0	0
E24	:	transfer me	1	2	2	0	0	0	0	0	1	1
E25	:	transfer pn	0	0	0	0	0	0	0	0	0	1

The failure-component similarity matrix indicates the similarity of the failure modes with respect to the components that the failure modes have in common. The diagonal simply gives '1.0' since the failure mode has the same components common with itself.

Application of Similarity Matrices to the Engine and Power Train of a Rotorcraft

The similarity matrices are derived using the normalized matrices $\overline{\mathbf{EC}}$, $\overline{\mathbf{CF}}_C$, and $\overline{\mathbf{CF}}_F$ derived from the function-component and component-failure mode matrices constructed earlier. The normalized matrix $\overline{\mathbf{EC}}$ was computed and is presented in Table 7. The similarity matrix of the component-function matrix, $\hat{\Lambda}_{CE}$, was also computed and is presented in Table 8. The component-function similarity matrix, $\hat{\Lambda}_{CE}$, communicates that components C_{18} and C_7 (nozzle and compressor wheel) are similar in function and C_{18} and C_{16} (nozzle and impeller) are similar in function when one is projected onto the other. The following groups of components have complete similarity (indicated by 1.0) with respect to

functionality: C_{16} and C_7 (impeller and compressor wheel); C_{23} and C_1 (Pressure control line and air discharge tubes); C_{22} and C_3 (plastic lining and bleed valve); C_{17} and C_5 (mount and compressor case); C_{28} and C_{27} (spur adapter gearshaft and shaft); and C_{11} and C_{19} (fire wall and nozzle shield). This indicates that some of these components can be redesigned with influence from the design of similar components in order to reduce or eliminate particular failure modes.

The components can be examined for common failure modes by examining the component-failure similarity matrix $\hat{\Lambda}_{CF}$. First the component-failure mode matrix's rows (components) are normalized. Only the components that experience a failure mode are present in the normalized matrix $\overline{\mathbf{CF}}_C$, shown in Table 9. The component-failure mode similarity matrix is found in Table 10. Several components share the same failure modes. Components C_7 and C_{29} (compressor wheel and turbine wheel) share the same failure modes (fatigue, stress rapture, and thermal shock), which is indicated '1.0'. C_5 and C_{26} (compressor case and rotor) have the failure mode corrosion in common. C_7 and C_{29} (compressor wheel and turbine wheel) experienced the same failure modes (fatigue, stress rupture, and thermal shock) and have high similarity in function (see Table 8, $\hat{\lambda}_{CE_{7,29}} = 0.7$). Thermal shock is an odd failure mode for the compressor wheel since the combustion chamber is after the compressor wheel. From the report that the data was gathered, it is believed that the last stage of the compressor section experienced excessive heat transfer from the combustion section due to inadequate shielding, and the compressor wheel failed because it was not designed for this scenario.

Next, the component-failure mode matrix is normalized by its columns (failure modes). The normalized matrix, $\overline{\mathbf{CF}}_F$, is in Table 11. The failure-component similarity matrix $\hat{\Lambda}_{FC}$ is shown in Table 12. F_1 and F_2 (bonding failure and corrosion) have one common component (shaft). The failure modes F_8 and F_9 (stress rupture and thermal shock) share two common components (compressor wheel and turbine wheel). There are failure modes that have more components in common, but the failure modes occur for so many components that their weights are low when normalized to unity. Conversely, many combinations of failure modes that do not occur together are indicated by a value of "0".

Use as a Potential Design-Aid Tool

The similarity matrices provide information for possible replacements or redesign of certain characteristics for components. It also provides a way to search and rank component solutions that are similar in function and use design by analogy techniques to embody a design. The component-function similarity and component-failure similarity matrices identify possible component solutions that prevent potential failure modes. If, between functionally-similar components A and B (as determined by $\hat{\Lambda}_{CE}$), component B does not experience all of the same failure modes as component A (as

determined by $\hat{\Lambda}_{CF}$), then there is some characteristic of component B that could be incorporated into A to improve its performance.

Consider the components C_5 and C_{17} (compressor case and mount), which have complete similarity in functionality and do not share any common failure modes as seen from $\hat{\Lambda}_{CF}$. One of the two components could be used to redesign the other component by determining what characteristics in each component reduces or eliminates the failures modes experienced by the other component and incorporating this information into the new design. Also, for the components that share common failure modes and functionality, the solution for reducing or eliminating a failure mode for one component could most likely be applied to the other component. This could be the case for C_6 and C_4 (compressor mount and bolt), which have complete similarity and have the failure mode F_3 (fatigue) in common.

Finally, the failure-component similarity matrix $\hat{\Lambda}_{FC}$ gives a mathematical picture of possible interactions between two or more failure modes. The elements indicate failure mode combinations that occur between components. These interactions can be used to direct component remedies that will possibly eliminate more than one failure mode and avoid catastrophic failure. For the current FMEA and FTA techniques, this knowledge of failure modes occurring interactively would give designers a more complete list of the possible product failures to be investigated.

CONCLUSIONS AND FUTURE WORK

In this paper, the function-failure method was applied to the engine and power train systems of rotorcraft to provide further evidence of the links between the functionality of a component to the potential failures of that component. This method provides rotorcraft designers an analytical means to capture systematic tradeoffs and design or redesign decisions based on similarities, to prevent potential failure modes. This method was applied earlier to a simple example using a rotating machinery test rig, to illustrate the potential of this method [6]. The purpose of the function-failure method is to aid NASA in the design of their high-risk aerospace endeavors, where safety is a high priority when failures can lead to fatal accidents. Severity was not incorporated into the data, because in the manner that the data was gathered all failures were equally severe in that they all caused engine and power train failure and an accident to occur. In the application of the method in this paper, actual failure data was gathered from NTSB (National Transportation Safety Board) reports and incorporated into the component-failure matrix, **CF**.

For future work, other areas of collecting failure data could give a more complete **CF** matrix. Possible places to acquire failure data would be from the records of failures from manufacturers of these aircraft and the records of failures logged by the military applications of these aircrafts. Furthermore, a method of consistent component naming will be introduced. This will provide a common generic way of classifying and representing the components in the mapping failure-function method proposed in this paper. This mapping of the failure-function method is currently being applied to a wide range of products [15]. The goal is to provide all this information stored in a repository that can be used by designers, and to expand this to as many products as possible.

The repetition of occurrence of failure modes for components over the time period for which the data was gathered was not used in this paper. In the future, the frequency of occurrence of a particular failure mode for a component will be incorporated to give more insight of the more probable potential failure modes.

REFERENCES

1. Collins, J.A., B.T. Hagan, and H.M. Bratt, *The failure-experience matrix: A useful design tool.* Journal of Engineering for Industry, 1976. August: p. 1074-1079.
2. Mitchell, J.S., *Introduction to machinery analysis and monitoring.* 2nd ed. 1993: PennWell Books.
3. Carter, A.D.S., *Mechanical Reliability and Design.* 1997: John Wiley & Sons.
4. Smith, J.D., *Gear Noise and Vibration.* 1999: Marcel Dekker.
5. McAdams, D. and K.L. Wood. *Quantitative measures for design by analogy.* in *Proceedings of the Design Engineering Technical Conferences.* 2000. Baltimore, MD: ASME.
6. Tumer, I.Y. and R.B. Stone. *Analytical method for mapping function to failure during high-risk component development.* in *Proceedings of the Design Engineering Technical Conferences.* 2001. Pittsburgh, PA: ASME.
7. Stone, R.B., K.L. Wood, and R.H. Crawford. *Product architecture development with quantitative functional models.* in *Proceedings of the Design Engineering Technical Conferences.* 1999. Las Vegas, NV: ASME.
8. Stone, R.B., K.L. Wood, and R.H. Crawford, *Using quantitative functional models to develop product architectures.* Design Studies, 2000. **21**(3): p. 239-260.
9. Harris, F.D., E.F. Kasper, and L.E. Iseler, *U.S. Civil Rotorcraft Accidents, 1963 through 1997.* 2000.
10. Shafer, J., *Fundamentals of Helicopter Maintenance.* 1980, Basin, WY: Aviation Maintenance Publishers, Inc.
11. Huff, E.M., I.Y. Tumer, and M. Mosher. *An experimental comparison of transmission vibration responses from OH58C and AH1 helicopters.* in *American Helicopter Society's 57th Annual Forum.* 2001. Washington, D.C.
12. Arterburn, D.R., *Personal Communications.* 2001: NASA Ames Research Center.
13. NTSB, *National Transportation Safety Board, http://www.ntsb.gov.* 2001.
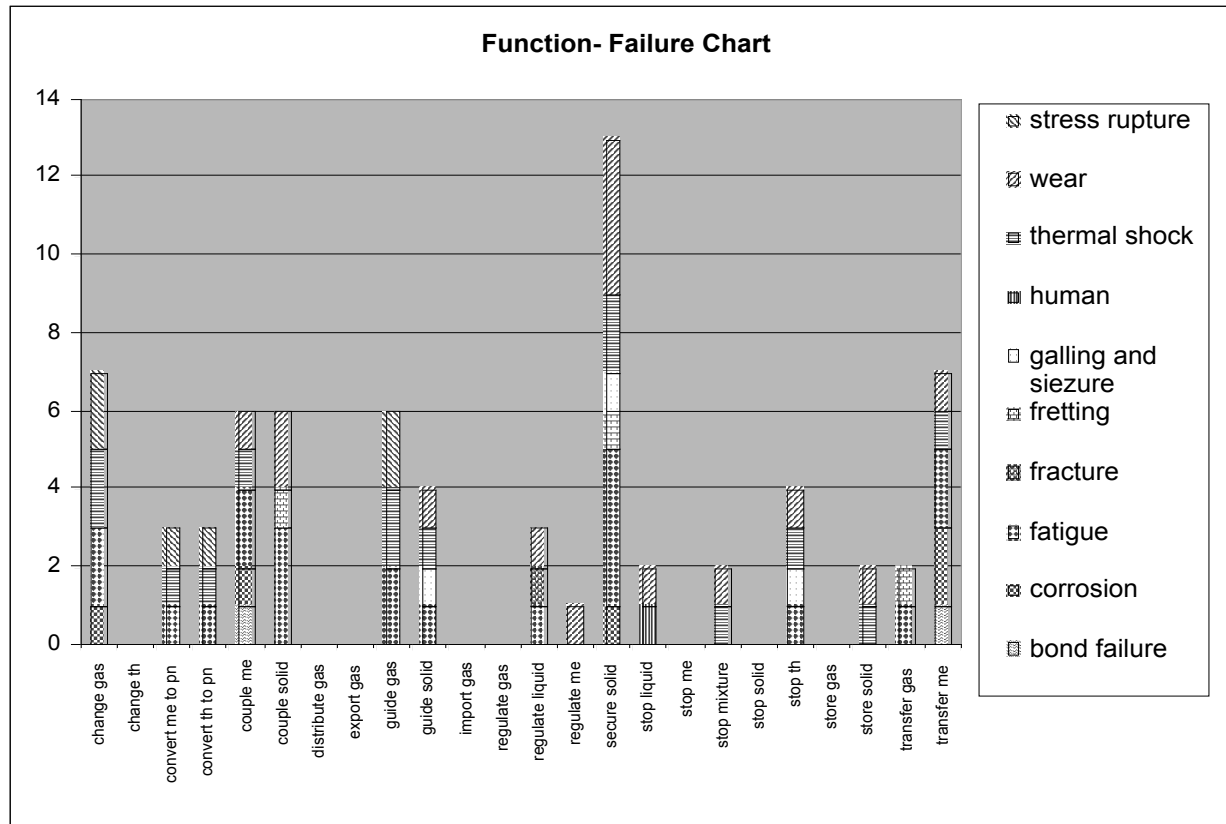
Figure 4. Function-failure mode chart

14. Stone, R.B. and K.L. Wood, *Development of a Functional Basis for Design.* Journal of Mechanical Design, 2000. 122(December): p. 359-370.
15. Arunajadai, S.G., R.B. Stone, and I.Y. Tumer. *A framework for creating a function-based design tool for failure mode identification.* in *Proceedings of the Design Engineering Technical Conferences* DETC2002/DTM34018. 2002.
16. Hirtz, J., Stone, R., McAdams, D., Szykman, S. and Wood, K., 2002, "A Functional Basis for Engineering Design: Reconciling and Evolving Previous Efforts," Research in Engineering Design, 13(2):65-82.
17. Little, A., D. McAdams and K. Wood. *Functional Analysis: A Fundamental Empirical Study for Reverse Engineering, Benchmarking and Redesign* in *Proceedings of the Design Engineering Technical Conferences.* 1997. Sacramento, CA: ASME.
18. McAdams, D., R. Stone, and K. Wood. *Functional Interdependence and Product Similarity Based on Customer Needs.* Research in Engineering Design, 1999. 11(1):p.1-19.

Table 7. The normalized matrix, $\overline{\mathbf{EC}}$

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18	C19	C20	C21	C22	C23	C24	C25	C26	C27	C28	C29
E1	0.0	0.0	0.0	0.0	0.0	0.0	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.6	0.0	0.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.7	0.0	0.0	0.6
E2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
E3	0.0	0.0	0.0	0.0	0.0	0.0	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
E4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.6
E5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.7	0.7	0.0
E6	0.0	0.0	0.0	0.7	0.0	0.7	0.0	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
E7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.7	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
E8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.7	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
E9	0.0	0.0	0.0	0.0	0.0	0.0	0.6	0.0	0.0	0.0	0.0	0.0	0.7	0.5	0.0	0.6	0.0	0.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.7	0.0	0.0	0.6
E10	0.0	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
E11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
E12	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
E13	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
E14	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.0
E15	0.0	0.6	0.0	0.7	1.0	0.7	0.0	0.0	0.0	0.0	0.0	0.0	0.7	0.5	0.0	0.6	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.7	0.7	0.0	0.0	0.0
E16	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
E17	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
E18	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
E19	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
E20	0.0	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
E21	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
E22	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
E23	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
E24	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.7	0.7	0.7	0.0
E25	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 8. The component-function similarity matrix

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18	C19	C20	C21	C22	C23	C24	C25	C26	C27	C28	C29
C1	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0
C2	0.0	1.0	0.0	0.4	0.6	0.4	0.0	0.0	0.0	0.0	0.6	0.4	0.3	0.0	0.3	0.0	0.6	0.0	0.6	0.0	0.0	0.0	0.0	0.4	0.4	0.0	0.0	0.0	0.0
C3	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
C4	0.0	0.4	0.0	1.0	0.7	1.0	0.0	0.4	0.0	0.0	0.0	0.5	0.4	0.0	0.4	0.0	0.7	0.0	0.0	0.0	0.0	0.0	0.0	0.5	0.5	0.0	0.0	0.0	0.0
C5	0.0	0.6	0.0	0.7	1.0	0.7	0.0	0.0	0.0	0.0	0.0	0.7	0.5	0.0	0.6	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.7	0.7	0.0	0.0	0.0
C6	0.0	0.4	0.0	1.0	0.7	1.0	0.0	0.4	0.0	0.0	0.0	0.5	0.4	0.0	0.4	0.0	0.7	0.0	0.0	0.0	0.0	0.0	0.0	0.5	0.5	0.0	0.0	0.0	0.0
C7	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.4	0.3	0.0	0.0	1.0	0.0	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.4	0.4	0.0	0.0	0.7
C8	0.0	0.0	0.0	0.4	0.0	0.4	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.4	0.8	0.8	0.0
C9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
C10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.8	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
C11	0.0	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
C12	0.0	0.4	0.0	0.5	0.7	0.5	0.4	0.0	0.0	0.0	0.0	1.0	0.7	0.0	0.4	0.4	0.7	0.5	0.0	0.0	0.0	0.0	0.0	0.5	1.0	0.0	0.0	0.0	0.4
C13	0.0	0.3	0.0	0.4	0.5	0.4	0.3	0.0	0.0	0.0	0.0	0.7	1.0	0.0	0.3	0.3	0.5	0.4	0.0	0.0	0.0	0.0	0.0	0.4	0.7	0.0	0.0	0.0	0.3
C14	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
C15	0.0	0.3	0.0	0.4	0.6	0.4	0.0	0.0	0.0	0.0	0.0	0.4	0.3	0.0	1.0	0.0	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.4	0.4	0.0	0.0	0.0	0.0
C16	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.4	0.3	0.0	0.0	1.0	0.0	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.4	0.4	0.0	0.0	0.7
C17	0.0	0.6	0.0	0.7	1.0	0.7	0.0	0.0	0.0	0.0	0.0	0.7	0.5	0.0	0.6	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.7	0.7	0.0	0.0	0.0
C18	0.0	0.0	0.0	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.0	0.5	0.4	0.0	0.0	0.8	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.5	0.5	0.0	0.0	0.8
C19	0.0	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
C20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
C21	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
C22	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
C23	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0
C24	0.0	0.4	0.0	0.5	0.7	0.5	0.0	0.0	0.0	0.0	0.0	0.5	0.4	0.0	0.4	0.0	0.7	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.5	0.0	0.0	0.0	0.0
C25	0.0	0.4	0.0	0.5	0.7	0.5	0.4	0.0	0.0	0.0	0.0	1.0	0.7	0.0	0.4	0.4	0.7	0.5	0.0	0.0	0.0	0.0	0.0	0.5	1.0	0.0	0.0	0.0	0.4
C26	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.4	0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.4	0.0	0.5	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.5	0.5	0.4
C27	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.5	1.0	1.0	0.0
C28	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.5	1.0	1.0	0.0
C29	0.0	0.0	0.0	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.0	0.4	0.3	0.0	0.0	0.7	0.0	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.4	0.4	0.0	0.0	1.0

Table 9. Normalized component-failure mode matrix by rows, $\overline{\text{CF}}_C$

	F1	F2	F3	F4	F5	F6	F7	F8	F9
C2 :	0.00	0.00	0.50	0.00	0.00	0.50	0.00	0.00	0.50
C4 :	0.00	0.00	0.58	0.00	0.58	0.00	0.00	0.00	0.00
C5 :	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C6 :	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
C7 :	0.00	0.00	0.58	0.00	0.00	0.00	0.00	0.58	0.58
C8 :	0.00	0.00	0.71	0.00	0.00	0.00	0.00	0.00	0.00
C14 :	0.00	0.00	0.58	0.58	0.00	0.00	0.00	0.00	0.00
C15 :	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.71
C17 :	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
C20 :	0.00	0.00	0.00	0.00	0.00	0.00	0.71	0.00	0.00
C21 :	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C23 :	0.00	0.00	0.71	0.00	0.71	0.00	0.00	0.00	0.00
C24 :	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C26 :	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C27 :	0.58	0.58	0.00	0.00	0.00	0.00	0.00	0.00	0.58
C28 :	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
C29 :	0.00	0.00	0.58	0.00	0.00	0.00	0.00	0.58	0.58

Table 11. Normalized component-failure mode matrix by columns, $\overline{\text{CF}}_F$

	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10
C1 :	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
C2 :	0.0	0.0	0.3	0.0	0.0	1.0	0.0	0.0	0.4	0.4
C3 :	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
C4 :	0.0	0.0	0.3	0.0	0.7	0.0	0.0	0.0	0.0	0.4
C5 :	0.0	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
C6 :	0.0	0.0	0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0
C7 :	0.0	0.0	0.3	0.0	0.0	0.0	0.0	0.7	0.4	0.0
C8 :	0.0	0.0	0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.4
C9 :	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
C10 :	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
C11 :	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
C12 :	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
C13 :	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
C14 :	0.0	0.0	0.3	1.0	0.0	0.0	0.0	0.0	0.0	0.4
C15 :	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.4	0.4
C16 :	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
C17 :	0.0	0.0	0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0
C18 :	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
C19 :	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
C20 :	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.4
C21 :	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.4
C22 :	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
C23 :	0.0	0.0	0.3	0.0	0.7	0.0	0.0	0.0	0.0	0.0
C24 :	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.4
C25 :	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
C26 :	0.0	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
C27 :	1.0	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.4	0.0
C28 :	0.0	0.0	0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0
C29 :	0.0	0.0	0.3	0.0	0.0	0.0	0.0	0.7	0.4	0.0

Table 10. The component-failure similarity matrix, $\hat{\Lambda}_{CF}$

	C2	C4	C5	C6	C7	C8	C14	C15	C17	C20	C21	C23	C24	C26	C27	C28	C29
C2 :	1.0	0.6	0.0	0.5	0.6	0.7	0.6	0.7	0.5	0.4	0.5	0.4	0.5	0.0	0.3	0.5	0.6
C4 :	0.6	1.0	0.0	0.6	0.3	0.8	0.7	0.4	0.6	0.4	0.6	0.8	0.6	0.0	0.0	0.6	0.3
C5 :	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.6	0.0	0.0
C6 :	0.5	0.6	0.0	1.0	0.6	0.7	0.6	0.0	1.0	0.0	0.0	0.7	0.0	0.0	0.0	1.0	0.6
C7 :	0.6	0.3	0.0	0.6	1.0	0.4	0.3	0.4	0.6	0.0	0.0	0.4	0.0	0.0	0.3	0.6	1.0
C8 :	0.7	0.8	0.0	0.7	0.4	1.0	0.8	0.5	0.7	0.5	0.7	0.5	0.7	0.0	0.0	0.7	0.4
C14 :	0.6	0.7	0.0	0.6	0.3	0.8	1.0	0.4	0.6	0.4	0.6	0.4	0.6	0.0	0.0	0.6	0.3
C15 :	0.7	0.4	0.0	0.0	0.4	0.5	0.4	1.0	0.0	0.5	0.7	0.0	0.7	0.0	0.4	0.0	0.4
C17 :	0.5	0.6	0.0	1.0	0.6	0.7	0.6	0.0	1.0	0.0	0.0	0.7	0.0	0.0	0.0	1.0	0.6
C20 :	0.4	0.4	0.0	0.0	0.0	0.5	0.4	0.5	0.0	1.0	0.7	0.0	0.7	0.0	0.0	0.0	0.0
C21 :	0.5	0.6	0.0	0.0	0.0	0.7	0.6	0.7	0.0	0.7	1.0	0.0	1.0	0.0	0.0	0.0	0.0
C23 :	0.4	0.8	0.0	0.7	0.4	0.5	0.4	0.0	0.7	0.0	0.0	1.0	0.0	0.0	0.0	0.7	0.4
C24 :	0.5	0.6	0.0	0.0	0.0	0.7	0.6	0.7	0.0	0.7	1.0	0.0	1.0	0.0	0.0	0.0	0.0
C26 :	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.6	0.0	0.0
C27 :	0.3	0.0	0.6	0.0	0.3	0.0	0.0	0.4	0.0	0.0	0.0	0.0	0.0	0.6	1.0	0.0	0.3
C28 :	0.5	0.6	0.0	1.0	0.6	0.7	0.6	0.0	1.0	0.0	0.0	0.7	0.0	0.0	0.0	1.0	0.6
C29 :	0.6	0.3	0.0	0.6	1.0	0.4	0.3	0.4	0.6	0.0	0.0	0.4	0.0	0.0	0.3	0.6	1.0

Table 12. The failure-component matrix, $\hat{\Lambda}_{FC}$

	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10
F1 :	1.0	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.4	0.0
F2 :	0.6	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.3	0.0
F3 :	0.0	0.0	1.0	0.3	0.4	0.3	0.0	0.4	0.4	0.4
F4 :	0.0	0.0	0.3	1.0	0.0	0.0	0.0	0.0	0.0	0.4
F5 :	0.0	0.0	0.4	0.0	1.0	0.0	0.0	0.0	0.0	0.3
F6 :	0.0	0.0	0.3	0.0	0.0	1.0	0.0	0.0	0.4	0.4
F7 :	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.4
F8 :	0.0	0.0	0.4	0.0	0.0	0.0	0.0	1.0	0.6	0.0
F9 :	0.4	0.3	0.4	0.0	0.0	0.4	0.0	0.6	1.0	0.3
F10 :	0.0	0.0	0.4	0.4	0.3	0.4	0.4	0.0	0.3	1.0